TEHNOLOGIC

SUBIECTUL I

1. $\left(\frac{1}{2}+\frac{1}{5}\right)\cdot\frac{20}{7}=\frac{7}{10}\cdot\frac{20}{7}=2$

2. $A(a,0)\in G_f \Leftrightarrow f(a)=0 \Leftrightarrow a-2=0 \Leftrightarrow a=2$

3. $\sqrt{x+3}=4$; $x+3\geq 0 \Leftrightarrow x\geq -3 \Leftrightarrow x\in[-3;+\infty)$

$\sqrt{x+3}=4 \Leftrightarrow x+3=4^2 \Leftrightarrow x=16-3 \Leftrightarrow x=13\in[-3;+\infty)$

deci $x=13$

4. Multiplii de 15 din M sunt : 30, 60, 90

$p=\frac{\text{nr. cazuri favorabile}}{\text{nr. cazuri posibile}}=\frac{3}{9}=\frac{1}{3}$

deci probabilitatea este $\frac{1}{3}$

5. Fie M mijloc (AB) $\Rightarrow x_M=\frac{x_A+x_B}{2}=\frac{4+4}{2}=4$

$y_M=\frac{y_A+y_B}{2}=\frac{2+6}{2}=4$

deci $M(4;4)$

6. $x\in\left(0;\frac{\pi}{2}\right)\Rightarrow \cos x>0$

$\sin^2 x+\cos^2 x=1$; $\forall x\in\mathbb{R} \Leftrightarrow \sin^2 x+\frac{25}{169}=1$

$\Leftrightarrow \sin^2 x=1-\frac{25}{169} \Leftrightarrow \sin^2 x=\frac{144}{169}$ | $\Rightarrow \sin x=\frac{12}{13}$

$\sin x>0$

SUBIECTUL II

1a) $\det A=\begin{vmatrix}1 & 2\\ 3 & 4\end{vmatrix}=1\cdot 4-2\cdot 3=4-6=-2$

b) $A+B=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}+\begin{pmatrix}4 & 3\\ 2 & 1\end{pmatrix}=\begin{pmatrix}5 & 5\\ 5 & 5\end{pmatrix}=5\cdot\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}=$

$=5\cdot C$

II 1.c) $AB+BA+4I_2 =$

$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 4 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$

$= \begin{pmatrix} 4+4 & 3+2 \\ 12+8 & 9+4 \end{pmatrix} + \begin{pmatrix} 4+9 & 8+12 \\ 2+3 & 4+4 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} =$

$= \begin{pmatrix} 25 & 25 \\ 25 & 25 \end{pmatrix} = 25 \cdot \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 25 \cdot C$

2. a) $5 \circ (-4) = 5 \cdot (-4) + 4 \cdot 5 + 4 \cdot (-4) + 12 =$

$= -20 + 20 - 16 + 12 = -4$

b) $(x+4)(y+4) - 4 = xy + 4x + 4y + 16 - 4 =$

$= xy + 4x + 4y + 12 = x \circ y$

c) $x \circ x = (x+4)(x+4) - 4 = (x+4)^2 - 4$

$x \circ x = x \Leftrightarrow (x+4)^2 - 4 = x \Leftrightarrow (x+4)^2 - (4+x) = 0$

$\Leftrightarrow (x+4)(x+4-1) = 0 \Rightarrow x+4 = 0$ sau $x+3 = 0$

$\Rightarrow x = -4$ sau $x = -3$

SUBIECTUL III

1. a) $f'(x) = 6x^2 + 6x = 6x(x+1)$

b) $\lim\limits_{x \to +\infty} \dfrac{f'(x)}{f(x) - 2x^3} = \lim\limits_{x \to +\infty} \dfrac{6x^2 + 6x}{3x^2 + 5} =$

$= \lim\limits_{x \to +\infty} \dfrac{x^2\left(6 + \frac{6}{x}\right)}{x^2\left(3 + \frac{5}{x}\right)} = \lim\limits_{x \to +\infty} \dfrac{6 + \frac{6}{x}}{3 + \frac{5}{x}} = \dfrac{6}{3} = 2$

pentru că $\lim\limits_{x \to +\infty} \dfrac{6}{x} = \lim\limits_{x \to +\infty} \dfrac{5}{x} = 0$

III 1 a) $f'(x)=0 \Rightarrow x(x+1)=0 \Rightarrow x=0$ sau $x=-1$

x	$-\infty$		-1		0		$+\infty$
$f'(x)$	+ + +	0	− − −	0	+ + + +		

deci $x \in (-\infty,-1) \Rightarrow f'(x)>0 \Rightarrow f$ strict crescătoare pe $(-\infty,-1)$

deci $x \in (-1,0) \Rightarrow f'(x)<0 \Rightarrow f$ strict descresc. pe $(-1,0)$

deci $x \in (0,+\infty) \Rightarrow f'(x)>0 \Rightarrow f$ strict cresc. pe $(0,+\infty)$

2. a) $\int_1^2 (f(x)-3x^2)\,dx = \int_1^2 4x^3\,dx = x^4\Big|_1^2 = 2^4-1^4=$

$=16-1=15$

b) $\int f(x)\,dx = x^4+x^3+k$, $k \in \mathbb{R}$

deci $F(x)=x^4+x^3+k$ / $F(1)=2015$ $\Rightarrow 1+1+k=2015 \Rightarrow k=2013$

$\Rightarrow F(x)=x^4+x^3+2013$

c) $\int_1^n \frac{f(x)}{x^2}\,dx = 9 \Leftrightarrow \int_1^n (4x+3)\,dx = 9 \Rightarrow$

$\Leftrightarrow (2x^2+3x)\Big|_1^n = 9 \Leftrightarrow 2n^2+3n-(2+3)=9 \Rightarrow$

$\Leftrightarrow 2n^2+3n=14 \Leftrightarrow n(2n+3)=14$, $n \in \mathbb{N}$ / $\Leftrightarrow n$ divide pe 14

$\Rightarrow n \in \{1,2,7,14\}$

Prin verificare se obține $n=2$